University of Nevada, Reno

Development of a Nonlinear Binary Programming Model for Intersection Improvement Prioritization

A thesis submitted in partial fulfillment of the
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by

David M. Hutchinson

Dr. Hao Xu/Thesis Advisor

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DAVID M. HUTCHINSON

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Hao Xu, Ph.D., Advisor

Zong Tian, Ph.D., Committee Member

Thomas Quint, Ph.D., Graduate School Representative

David W. Zeh, Ph.D., Dean, Graduate School

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ABSTRACT

A model is presented that utilizes nonlinear binary mathematical programming and the predictive crash methodology found in the AASHTO Highway Safety Manual to prioritize the selection of intersection safety projects. This research improves upon methods employed by prioritization tools such as the Interactive Highway Data Safety Design Model (IHSDM) and AASHTO Safety Analyst by the ability to implement multiple countermeasures at individual intersections. This expansion is advantageous in that it that it allows greater flexibility in determining mitigation strategies. For real life applications, implementation of multiple countermeasures can also decrease overall project expense by reducing construction mobilization and oversight costs. A sample set of intersections is analyzed using the nonlinear binary method and results are compared to a similar model utilizing a single countermeasure or a single set of predetermined countermeasures.

Keywords: Nonlinear Programming, Intersection Safety, Highway Safety Manual, Crash Modification Factors, Integer Programming

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

1. INTRODUCTION

Intersection traffic crashes are a hidden global pandemic that impact a countless number of lives each year. Intersection and intersection-related crashes make up 23 percent of all fatal crashes and more than 50 percent of combined fatal and injury crashes (FHWA, 2014). Along with loss of life and injury, these crashes create substantial property damage and decreased mobility that have enormous societal impacts. Despite improved intersection design and more sophisticated applications of traffic engineering measures, intersection crashes remain a serious concern. Intersection safety presents a unique challenge in that intersections are planned points of conflict in any roadway system. Increased conflict potential and confusing geometric characteristics provide many potential mitigation strategies that can be employed for crash reduction at these facilities.

The AASTHTO Highway Safety Manual (HSM) provides guidance for the estimation of crashes and the processes used for roadway network screening and diagnosis of safety deficiencies. Chapter 8 of the HSM introduces mathematical methods used for the prioritization of safety improvements for highway safety programs. There are three specific optimization methods that can potentially be used for prioritization of safety projects. These are:

- Linear programming (LP) optimization
- Integer programming (IP) optimization
- Dynamic programming (DP) optimization

While the methods detailed in the HSM provide a strong analytical framework for project prioritization, they lack the ability to easily accommodate the implementation of multiple countermeasures at individual locations. This is because the methods described use linear programming as the basis for analysis. The multiplicative nature of crash reduction factors result in objective functions that contain products of decision variables, making nonlinear integer programming the most suitable method of problem solution. While it is possible to create a linear model that will accommodate the selection of multiple improvements at individual locations, the effort required is substantially greater to develop the model description. This is because decision variables need to be created for every possible configuration of countermeasures in order for the model to retain linearity. A linear programming model utilizing 30 countermeasures at 20 locations requires 90,520 decision variables for a solution, while a nonlinear formulation would require 600 variables. The number of constraints necessary for the description of a linear model is also substantially larger. While nonlinear methods generally require more processing time, the time spent coding the program will be substantially less. The aim of this study is to formulate a model that utilizes nonlinear programming to optimize the selection of countermeasures at a set of intersections.

2. LITERATURE REVIEW

While used extensively for pavement management, integer programming was utilized infrequently for traffic safety project utilization until the release of the AASHTO Highway Safety Manual in 2010. Integer programming methods have been used for traffic safety analysis as far back as 1979. In the paper “Assessment of techniques for cost-effectiveness of highway countermeasures" prepared for the FHWA by the Texas Transportation Institute (William F. McFarland, 1979), the authors utilize the following integer program:

$$\max \sum_{j=1}^{J} b_i X_{ij}$$

$$\text{s.t.} \sum_{i=1}^{n} C_i X_i \le B$$

$$\sum_{k=1}^{k} X_{ik} = 1 \qquad \textit{where } k \in G_i$$

$$X_i = 0, 1$$

where b_i is the benefit coefficient for X_i, C_i is the cost coefficient for X_i and B is the total amount of the resource available. G_i is the generalized upper bound (G.U.B.) constraint for variable X_i. If $X_i = 1$, X_i has been chosen for inclusion in the solution at a profit or benefit of b_i and a use of C_i of the constrained resource. If $X_i = 0$, then the variable X_i has not been chosen for inclusion in the solution. There is a G.U.B. constraint for each location considered; therefore, the number of variables associated with that constraint correspond to location alternatives. This method is limited because it only considers the most effective single improvement for each site. The model introduced in the Texas Transportation Institute was discussed in various paper was restated in multiple publications, including the 1983 Joint Highway Research Project publication "Priority Setting of Highway Improvement Projects" authored by researchers at Purdue University (Sinha, 1983). In the 2004 TRB paper titled "Development of a Safety Resource-Allocation Model in Michigan," authors Kar and Datta used a safety performance index (SPI) applied to a linear programming model to prioritize safety projects (Kar, 2004). The focus of crash reduction for this model includes alcohol related and unrestrained drivers.

The SPI for an area can be calculated as follows:

$$\text{SPI} = W_F + PRC + FRC + IRC + SRC$$

where

W_F = weighted frequency = $(PCF + w_1 * FCF + w_2 * ICF + w_3 * SCF)/1000$,
PCF = PDO crash frequency,
FCF = fatal crash frequency,
ICF = injury crash frequency,
SCF = special focus crash frequency,
w_1 = weighting factor for fatal crash frequency,
w_2 = weighting factor for injury crash frequency,
w_3 = weighting factor for special focus crash frequency,
PRC = PDO crash rate composite = (PDO crashes per 1,000 population + PDO crashes per 1,000 registered vehicles + PDO crashes per 10,000,000 VMT)/3,
FRC = fatal crash rate composite = (fatal crashes per 1,000 population + fatal crashes per 1,000 registered vehicles + fatal crashes per 10,000,000 VMT)/3,

Using SPI's as defined above, Karr and Datta defined the following optimization model:

Objective Function

The objective function is

$$\text{Maximize } Z = \sum(B_i * X_i)$$

where

X_i = decision variable = budget allocations to city and townships,

$$B_i = \text{coefficient} = \frac{(\text{SPI}_i * RC)}{\sum(\text{SPI}_i * RC)}$$

Where:

SPI_i = SPI for a city or township,
RC = average reduction in crashes = ΣRC_i/n,
RC_i = percent reduction in crashes for any project or program in a particular area, and
n = number of areas for which individual percent change in crashes is calculated.

Model constraints include:

- A resource allocation constraint which limits funding to agencies based on a predefined value. The use of this constraint prevents funds from being allocated to one single agency
- A constraint that limits funding allocated for alcohol related improvements to a predefined budget
- A constraint that limits funding allocated for driver-restraint related improvements to a predefined budget
- A non-negativity constraint that prohibits the decision variable X_i from values of less than zero.

Solution of this model provides funding allocations for programs aimed at reducing target crashes. This model differs from the one presented in this paper in that it is not used in the selection of specific improvements, but rather assumes a fixed crash reduction that increases the objective function as extra funding is applied. In 2012, the paper "Optimization Model for Allocating Resources for Highway Safety Improvement at Urban Intersections" introduced a resource allocation model for implementing safety improvement alternatives at urban intersections over a multi-year planning horizon (Mishra, 2014). The authors Mishra and Khasnabis present the model:

Maximize

$$Z = \sum_{n=1}^{N} \sum_{j=1}^{J} \sum_{i=1}^{I} \sum_{k_j=1}^{l_j} \left[f_i^n r_{i,j}^f c^f + m_i^n r_{i,j}^m c^m + p_i^n r_{i,j}^p c^p \right] z_{i,j}^{k_j,n} \qquad (1)$$

subject to

$$\sum_{n=1}^{N} \sum_{j=1}^{J} \sum_{i=1}^{I} \sum_{k_j=1}^{l_j} \left[\pi_{i,j}^n y_{i,j}^{k_j,n} + o_{i,j}^n x_{i,j}^{k_j,n} \right] \leq \sum_{n=1}^{N} b_n \qquad (2)$$

$$\sum_{j=1}^{J} \sum_{k_j=1}^{l_j} z_{i,j}^{k_j,n} \leq \sum_{j=1}^{J} \sum_{k_j=1}^{l_j} \hat{z}_{i,j}^{k_j,n} \qquad (3)$$

$$\sum_{j=1}^{J} \sum_{k_j=1}^{l_j} z_{i,j}^{k_j,n} \leq \sum_{j=1}^{J} \sum_{k_j=1}^{l_j} \gamma_{i,j}^{k_j,n} \qquad (4)$$

where

$$\sum_{j}^{J} \gamma_{i,j}^{k_j,n} \leq \begin{cases} \sum_j^J y_{i,j}^{k_j,n} + \sum_j^J x_{i,j}^{k_j,n} & , \forall k_j > 0 \\ 0, & \text{Otherwise} \end{cases} \qquad (5)$$

$$\sum_{j}^{J} y_{i,j}^{k_j,n} \leq \begin{cases} 1, & k_j = 1 \\ 0, & \text{Otherwise} \end{cases} \qquad (6)$$

$$\sum_{j}^{J} x_{i,j}^{k_j,n} \leq \begin{cases} 1, & 1 < k_j \leq l_j \\ 0, & \text{Otherwise} \end{cases} \qquad (7)$$

$$z_{i,j}^{k_j,n}, \gamma_{i,j}^n, x_{i,j}^{k_j,n}, y_{i,j}^{k_j,n} \geq 0, \forall i, j, k_j \qquad (8)$$

Where:

b_n = Allocated budget ($) in analysis year *n*

c^f = Cost of fatal crash (*f*) in year *n*

c^m = Cost of injury crash (*m*) in year *n*

c^p = Cost of property damage (*p*) crash in year *n*

f_i^n = Expected number of fatal crashes for location *i* analysis period *n*

m_i^n = Expected number of injury crashes for location *i* analysis period *n*

p_i^n = Expected number of property damage crashes for location *i* analysis period *n*

$r^f_{i,j}$ = Crash reduction factor for fatal crashes for alternative *j* chosen at location *i*

$r^m_{i,j}$ = Crash reduction factor for injury crashes for alternative *j* chosen at location *i*

$r^p_{i,j}$ = Crash reduction factor for property damage crashes for alternative *j* chosen at location

$z_{i,j}^{k_j,n}$ Binary decision variable.
=1 when an alternative *j* is chosen for location *i* for the analysis year *n*, the alternative is effective for k_j years after installation
= 0 Otherwise

$\hat{z}_{i,j}^{k_j,n}$ Binary decision variable.
=1 when an alternative *j* is suggested for location *i* for the analysis year *n*, the alternative is effective for k_j years after installation (before allocation)
=0, when an alternative *j* is not suitable for location *i*

$x_{i,j}^{k_j,n}$ An auxiliary binary decision variable exists only for an alternative after first year of implementation but before the service life
= 1 when a new alternative *j* is implemented at location *i* for the analysis year *n*, and is effective for k_j years after installation, where $1 < k_j \leq l_j$
= 0 Otherwise

$y_{i,j}^{k_j,n}$ An auxiliary binary decision variable for a new alternative implementation
= 1 when a new alternative *j* is implemented at location *i* for the analysis year *n*, the alternative is effective for the first year of installation, where k_j=1
= 0 Otherwise

$\gamma_{i,j}^{k_j,n}$ Number of alternatives allocated to location *i* in the year *n* (where type of alternative *j* may vary), and the alternatives remain effective for k_j years after installation till the end of service life

$\pi^n_{i,j}$ = Capital cost for alternative j implemented in location I in the analysis year j

$o^n_{i,j}$ = Operation and maintenance cost for alternative *j* implemented in location *i* in the analysis year *n*

This method seeks to maximize crash cost benefits through the implementation of safety countermeasures. This procedure is advantageous in that it allows multiple countermeasures to be selected at individual sites. The drawback to this method is that by requiring countermeasures to be implemented in different years, a safety program utilizing this method would incur extra mobilization and construction costs. Repeated mobilizations also have the potential to cause a greater disruption to mobility due to increased construction activity. In the 2016 paper "An optimization model for improving highway Safety," The authors utilize a multi-objective integer programming model for crash minimization on Wyoming roads. The model description consists of two objective functions and one set of constraints:

$$\begin{cases} \text{Minimize } \sum_{i=1}^{n} N_i \\ \text{Minimize } \sum_{i=1}^{n} N_{f\&I_i} \\ \text{Subject to } \left(\sum_{i=1}^{n} \text{safety improvement cost}_i * x_i \right) \leq \text{Budget} \\ x_i \in \{0,\ 1\} \end{cases}$$

where N_i and $N_{f\&Ii}$ represent the predicted crashes and fatal-and-injury crashes on road i, respectively. x_i is an integer equal to 1 if the project is selected and 0 if it is not selected. The best combination of safety improvement projects are selected using linear programming methods. This is a combinatorial optimization problem where one must select a collection of projects of minimum value while satisfying some constraint. The predicted crashes Ni is the crashes of the segment multiplied by the CRF if the segment is selected for improvements. This model is a multi-level optimization where two objective functions were considered. The AASHTO Safety Analyst tool utilizes its own method for the prioritization of safety improvements for utilization within an intersection or roadway segment network. Safety Analyst maximizes the net benefits across a transportation network subject to budget constraints. This method compares available alternatives for individual locations and selects the alternative or predefined group of alternatives that will result in the greatest crash reduction. The limitation of this method is that it does not allow for flexibility in the implementation of multiple countermeasures at individual locations unless the countermeasure combinations have been computed in advance of being utilized in the model.

3. AASHTO HIGHWAY SAFETY MANUAL METHODOLOGY

The HSM consists of tools, that when used together, form an effective process for the management of safety programs. The process consists of six steps detailed in Figure 1. The following methods and descriptions are taken directly from the AASHTO Highway Safety Manual (AASHTO, Highway Safety Manual, 2010), with additional descriptions added where necessary to correlate the HSM methods to the new method proposed in this thesis.

A. The Network Screening Process

The Network screening process identifies and ranks locations with the highest potential for enhanced safety. Sites selected during the process are studied in greater detail to find potential mitigation strategies for implementation. Network screening can also be used as a tool to implement a policy, such as prioritizing the

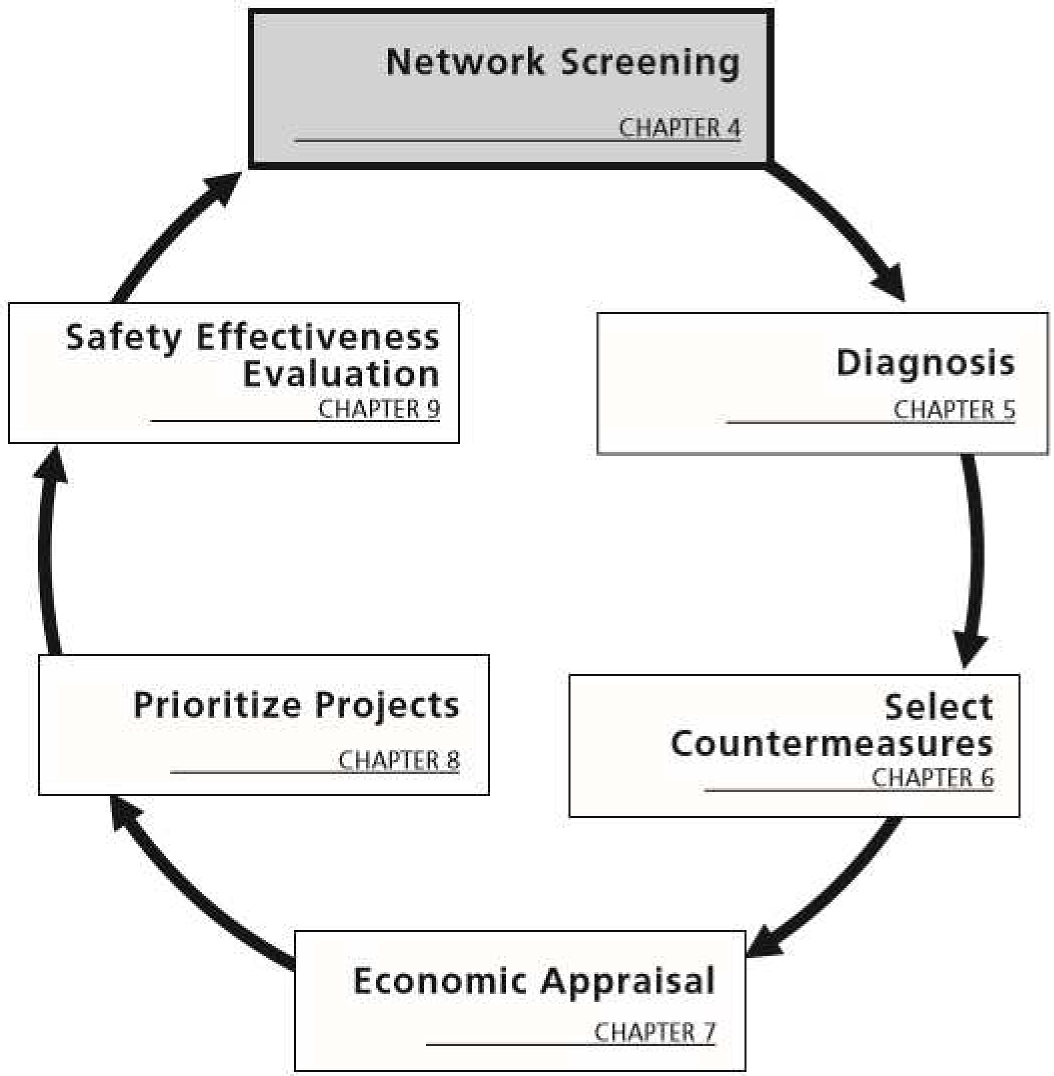

Figure 1. HSM Roadway Safety Management Process

replacement of non-standard guardrail statewide at sites with a high number of run-off-the-road crashes. As shown in figure 1, network screening is the first activity utilized in the cyclical Roadway Safety Management Process. Network screening analysis is performed by utilizing the steps detailed in figure 2.

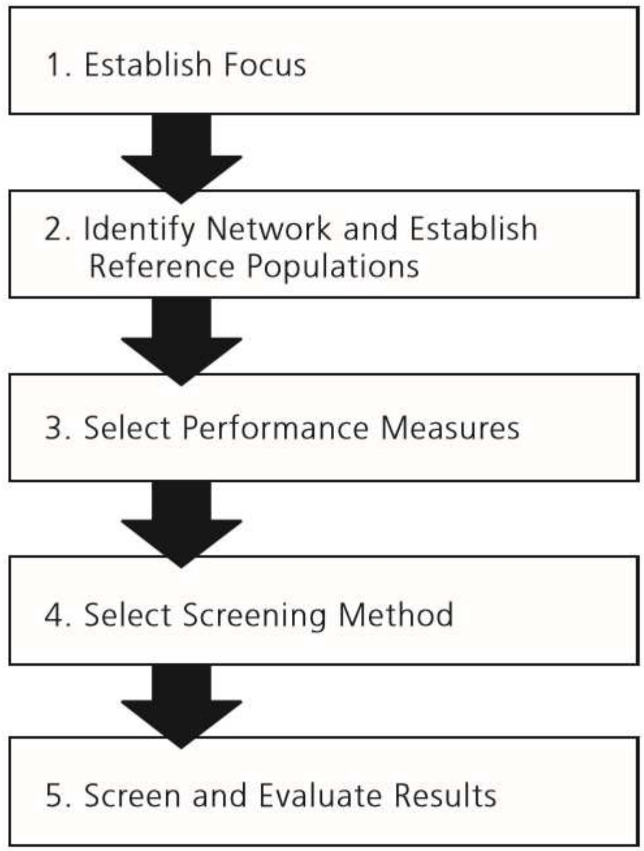

Figure 2. HSM Network Screening Process

The first step in the network process is to establish the focus of the analysis. Depending on the objective chosen, the focus could be the reduction of a particular crash type of the identification of sites where improvements have the greatest potential to reduce the greatest number of crashes. Once the focus has been identified, the network of sites for consideration is established and individual locations are organized into reference populations so that sites with similar characteristics are compared. Potential characteristics that can be used to establish reference populations for intersections include

- Traffic control (e.g., signalized, two-way or four-way stop control, yield control, roundabout);
- Number of approaches (e.g., three-leg or four-leg intersections);
- Cross-section (e.g., number of through lanes and turning lanes);
- Functional classification (e.g., arterial, collector, local);
- Area type (e.g., urban, suburban, rural);
- Traffic volume ranges (e.g., total entering volume (TEV, peak hour volumes, average annual daily traffic (AADT)); or
- Terrain (e.g., flat, rolling, mountainous)

The characteristics used for determination of reference populations can vary depending on network size, purpose, and the amount of detail known for the locations under consideration. Once the network and reference groups have been established, performance measures need to be defined to serve as the basis for network comparison. The HSM provides thirteen performance measures that can be chosen to serve as the basis for network screening, including:

- Average Crash Frequency
- Crash Rate
- Equivalent Property Damage Only (EPDO) Average Crash Frequency
- Relative Severity Index
- Critical Rate
- Excess Predicted Average Crash Frequency Using Method of Moments

- Level of Service Safety
- Excess Predicted Average Crash Frequency Using Safety Performance Functions (SPFs)
- Probability of Specific Crash Types Exceeding Threshold Proportion
- Excess Average Crash Frequency with EB adjustments
- EPDO Average Crash Frequency with EB Adjustment
- Excess Expected Crash Frequency with EB Adjustment
  (AASHTO, Highway Safety Manual, 2010)

Average Crash Frequency

The average crash frequency ranks locations with the greatest number of total crashes in a predefined time period. The benefit of using crash frequency as a performance measure is that it is simple to calculate. The limitations of using crash frequency are:

- The method does not account for regression-to-the-mean-bias
- Traffic volumes are not considered
- The method will not identify low-volume collision sites where simple cost-effective mitigating countermeasures could be easily applied
- Does not estimate a threshold to indicate sites experiencing more crashes than predicted for sites with similar characteristics

Crash Rate

To account for exposure, crash rate can be used as a performance measure. Crash rates are typically expressed as million entering vehicles (MEV) per intersection for the study period. The benefits of using crash rate are that it is simple and that it can be modified to account for severity if an EPDO or RSI-based crash count is used. The limitations include:

- Does not account for regression-to-the-mean bias
- Does not identify a threshold to indicate sites experiencing sites experiencing more crashes than predicted for sites with similar characteristics
- Comparisons cannot be made across sites with significantly different traffic volumes
- The method will mistakenly prioritize low volume, low collision sites

Equivalent Property Damage Only (EPDO) Average Crash Frequency

This performance measure assigns weighting factors to crashes by severity (fatal, injury, property damage only) to develop a score that combines crash frequency and severity. The weighting factors are determined by finding the crash cost of each severity level. Crash costs include direct and indirect costs. Direct costs could include: ambulance service, police and fire services, property damage, or insurance. Indirect costs include the value society would place on the pain and suffering and loss of life associated with the crash. The strengths of using EPDO equivalence are that it is simple and considers crash severity. The limitations of this method include:

- Does not account for regression-to-the-mean bias
- Does not identify a threshold to indicate sites experiencing more crashes than predicted for sites with similar characteristics
- Does not account for traffic volume
- May overemphasize locations with a low frequency of severe crashes depending on the weighting factors used

Relative Severity Index

Relative severity index determination involves assigning monetary costs associated with individual crash types and finding the total cost for each site under consideration. An average crash cost per site is determined for the reference population and locations with higher than average costs are considered for further analysis. Crash Costs used in the HSM for relative severity index are detailed in table 1. This method is simple and considers collision type and crash severity: however, it has the following limitations:

- Does not account for regression-to-the-mean bias
- May overemphasize locations with a small number of severe crashes depending on weighting factors used
- Does not account for traffic volume
- Will mistakenly prioritize low-volume, low-collision sites

Table 1. HSM Crash Cost by Collision Type

Crash Type	Crash Cost (2001 Dollars)
Rear-End, Signalized Intersection	$26,700
Rear-End, Unsignalized Intersection	$13,200
Sideswipe/Overtaking	$34,000
Angle, Signalized Intersection	$47,300
Angle, Unsignalized Intersection	$61,100
Pedestrian/Bike at an Intersection	$158,900
Head-On, Signalized Intersection	$24,100
Head-On, Unsignalized Intersection	$47,500
Fixed Object	$94,700
Other/Undefined	$55,100

Source: *Crash Cost Estimates by Maximum Police-Reported Injury Severity within Selected Crash Geometries*, FHWA-HRT-05-051, October 2005

Critical Rate

The critical rate performance measure compares the observed crash rate at each site to a calculated critical crash rate that is unique to each site. The critical crash rate is a threshold value that allows for a relative comparison among sites with similar characteristics. Locations that exceed their respective critical rates are flagged for further review. While critical rate does not account for regression-to-the-mean bias, it strengths include:

- Reduces exaggerated effect of sites with low volumes

- Considers variance in crash data
- Establishes a threshold for comparison

Excess Predicted Average Crash Frequency Using Method of Moments

This method adjusts a site's observed average crash frequency based on the variance in the crash data and average crash frequency for the reference population. A comparison is made between the site's adjusted crash frequency and that of the crash frequency for the reference population. This gives an indication of the potential for improvement which can be used as a basis ranking sites in a network. The strengths of this method include:

- Establishes a threshold of predicted performance for a site
- Considers variance in crash data
- Allows sites of all types to be ranked in one list
- Method concepts are similar to Empirical Bayes Methods

Limitation of using the method of moments method are:

- Does not account for regression-to-the-mean bias
- Does not account for traffic volume
- Some sites may be identified for further study because of unusually low frequency of non-target crash types
- Ranking results are influenced by reference populations; sites near boundaries of reference populations may be over-emphasized

Level of Service Safety (LOSS)

LOSS ranks sites using a qualitative assessment in which the observed crash count is compared to a predicted average crash frequency for the reference population under consideration (Allery, 2003) (Council, 2005) (Hauer, 1997) (AASHTO, Highway Safety Manual, 2010). LOSS classifications are developed based on the degree to which observed crash frequencies differ from the predicted average crash frequency. To account for variance, safety performance functions are used to establish the predicted average crash frequency used in LOSS analysis. While the effects of regression-to-the-mean analysis may still be present using this method, benefits of LOSS analysis include:

- Consideration of variance in crash data
- Traffic volumes are accounted for in analysis
- Method establishes a threshold for measuring potential to reduce crash frequency

Excess Predicted Average Crash Frequency Using Safety Performance Functions

This performance measure uses predicted average crash frequencies determined by safety performance functions (SPFs) as a basis of comparison to observed crash frequency at individual sites. When observed crashes exceed the predicted crashes as found using the SPF, the site is flagged for further analysis. This method accounts for traffic volume and provides a threshold for comparison. The main drawback to this method is that the effects of regression-to-the-mean bias may still be present in the results.

Probability of Specific Crash Types Exceeding Threshold Proportion

This method prioritizes sites based on the probability that the proportion of a particular crash type or severity is greater than a defined threshold proportion. Threshold proportions are determined for each population using data for reference populations. Strengths of this method are:

- It can be used as a diagnostic tool
- Considers variance in data
- Not affected by regression-to-the-mean bias

Limitations of this method include:

- Does not account for traffic volume
- Some sites may be identified for further study because of unusually low frequency of non-target crash types

Excess Proportions of Specific Crash Types

This method prioritizes sites based on the presence of excessive proportions of certain crash types. Threshold proportions of each crash are selected, and locations with the largest excess values are given the highest ranking. Drawbacks of this method are that traffic volumes are not accounted for and locations with low numbers of non-target crash frequencies have the potential to be flagged for further analysis when there is not a problem. The strengths of this method are:

- Consideration of variance in data
- Ability to be used as a diagnostic tool

- Analysis is not effected by regression-to-the-mean bias.

Expected Average Crash Frequency with Empirical Bayes (EB) Adjustment

This performance measure uses the Empirical Bayes method to weight observed and predicted crashes to calculate the expected crash frequency at a site. Sites are then ranked based on the expected average crash frequency. This method is beneficial in that it accounts for regression-to-the-mean bias. The drawback to its use is that it requires safety performance functions calibrated to local conditions for accurate results.

Equivalent Property Damage Only Average Crash Frequency with EB Adjustment

This method takes the equivalent property damage only (EPDO) method discussed above and adjusts it using the Empirical Bayes Method to estimated expected crashes at a particular location. The strengths of using this method include consideration of crash severity and that the regression-to-the-mean bias is mostly eliminated. The primary limitation is that the method may overemphasize locations with a small number of severe crashes depending on weighting factors used.

Excess Expected Average Crash Frequency with EB Adjustment

This method uses the Empirical Bayes method to estimate expected crashes at individual sites and compares the results to the predicted crashes found using a safety performance function. Sites with the greatest difference between the expected crash frequency and predicted crash frequency are considered for further analysis. While limited by the availability of safety performance functions for individual locations, this method accounts for regression-to-the mean bias and identifies a threshold to indicate sites experiencing sites experiencing more crashes than expected for sites with similar characteristics.

Once a measure of effectiveness is selected, sites are analyzed and results are used to diagnose potential mitigation strategies.

B. Diagnosis

Diagnostic analysis is performed to provide an understanding of crash patterns and physical characteristics before potential mitigation strategies are selected for implementation. Descriptive crash statistics and field conditions such as roadway characteristics, traffic conditions, and traveler behavior are used to determine crash conditions at individual sites. The expected outcome of the diagnostic review is an understanding of environmental conditions that have the potential to create crash patterns. The conclusions reached from this analysis are then used to help identify countermeasures that have the potential to enhance safety at locations flagged for consideration (AASHTO, Highway Safety Manual, 2010).

C. Select Countermeasures and Economic Appraisal

Once locations have been identified and diagnosed for safety deficiencies, the next step of the roadway safety management process is to select appropriate mitigation strategies for implementation within the network. The three main steps for selection of countermeasures are:

1. Identify factors contributing to the cause of crashes at the subject site;
2. Identify countermeasures which may address the contributing factors; and

3. Conduct benefit-cost analysis, if possible, to select preferred treatments.

Once countermeasures have been selected, an economic appraisal must be conducted. The two main objectives of the economic appraisal are:

1. Determine if a project is economically justified (i.e., the benefits are greater than the costs), and
2. Determine which project or alternative is the most cost-effective.

Two methods are presented in the HSM that can be used to conduct benefit-cost analysis to satisfy the first objective, namely the Net Present Value (NPV) method and the Benefit-Cost Ratio (BCR). The second objective is analyzed by comparing the cost-effectiveness of a countermeasure implementation project expressed as the annual cost per crash reduced for different countermeasure implementations. The methodology introduced later in this paper seeks to eliminate the need to choose a single countermeasure for implementation at an individual site. This results in higher crash reduction results and more flexibility in countermeasure selection.

D. Prioritize Projects

The HSM uses economic appraisal output from countermeasure selection to prioritize projects. Once countermeasures have been identified for implementation, the following procedures can be utilized to decide which mitigation strategies should be selected:

1. Ranking by economic effectiveness measures
2. Incremental benefit-cost analysis ranking

3. Optimization methods

Ranking by Economic Effectiveness Measures

This is the simplest method for project prioritization. It involves ranking projects by the following measures, including:

- Project costs,
- Monetary value of project benefits,
- Number of total crashes reduced,
- Number of fatal and incapacitating injury crashes reduced,
- Number of fatal and injury crashes reduced,
- Cost-effectiveness index, and
- Net present value (NPV)

While straightforward to calculate, these methods do not account for budget constraints and are too simple for situations with multiple competing priorities.

Incremental Benefit-Cost Analysis

This method generalizes benefit-cost analysis as follows:

1. Perform a benefit-cost ratio evaluation for each individual improvement project
2. Arrange projects with a BCR greater than 1.0 in increasing order based on their estimated cost. The project with the smallest cost is listed first.
3. Beginning at the top of the list, calculate the difference between the first and second project's benefits. Similarly calculate the difference

between the costs of the first and second projects. The differences between the benefits of the two projects and the costs of the two are used to compute the BCR for the incremental investment.

4. If the BCR for the incremental investment is greater than 1.0, the project with the higher cost is compared to the next project on the list. If the BCR for the incremental investment I less than 1.0, the project with the lower cost is compared to the next project in the list.
5. Repeat this process. The project selected in the last pairing is considered the best economic investment.

The process is repeated without projects previously determined to be the best economic investment until the ranking of every project is determined.

Optimization Methods

Optimization methods are employed to identify a set of projects that will maximize the benefits desired constrained to a given budget. It is assumed that all projects have been evaluated and found to be economically justified. The method chosen for implementation will be dependent on:

- The need to consider budget or other constraints, or both, within the prioritization, and
- The type of software accessible, which could be as a simple as a spreadsheet or as complex as specialized software designed for the method.

The Optimization methods detailed in the HSM include:

- Linear programming (LP) optimization
- Integer programming (IP) optimization
- Dynamic programming (DP) optimization

Linear Programming

Linear programming methods are mathematical optimization models that find the maximum/minimum value of an equation subject to constraints. The general form of a linear programming problem is expressed as:

Objective Function:

$$\text{Optimize:} \quad c_1x_1 = c_2x_2 + \ldots + c_nx_n$$

Constraint Equations:

$$\text{Subject to:} \quad a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \geq b_1$$

$$a_{21}x_1 + a_{22}x_2 + \ldots + a_{2n}x_n \geq b_2$$

$$\cdots \qquad \cdots \qquad \cdots$$

$$\cdots \qquad \cdots \qquad \cdots$$

$$a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \geq b_m$$

$$x_j \geq 0 \quad \text{for } j = 1,2,\ldots n$$

The objective function is the value to be optimized, while the constraints provide limits for potential solutions under consideration. Various methods can be utilized to solve linear programming problems, with the simplex method being the most common form of solution. Linear programming methods have been applied to traffic safety applications; however, no known applications of linear programming for project prioritization exist because integer programming is better suited for this type of application.

Integer Programming

Integer programming is a subset of linear programming that requires variables to be restricted to integer values. The general form of an integer programming problem can be expressed as:

$$\text{Maximize} \sum_{j=1}^{n} c_j x_j,$$

subject to:

$$\sum_{j=1}^{n} a_{ij} x_j = b_i \qquad (i = 1, 2, \ldots, m),$$

$$x_j \geq 0 \qquad (j = 1, 2, \ldots, n),$$

$$x_j \text{ integer} \qquad (\text{for some or all } j = 1, 2, \ldots, n).$$

Integer programming is useful in traffic safety optimization problems because most often the variables used are binary decision variables. A binary decision variable is used when "yes or no" decisions need to be made in an optimization problem. This is common in situations where project alternatives will or will not be implemented. Because of the binary nature of project decision making, integer programming has been implemented more widely than linear programming for highway safety applications. Similar to linear programming methods, integer programming problems include budget constraints and constraints that prohibit the implementation of projects deemed infeasible for a particular site. Integer programming problems can be solved analytically using cutting plane algorithms and branch-and-bound methods. Software packages such as Microsoft Excel and

Lindo are also available to solve integer programming problems. A general-purpose optimization tool based on integer programming is available in the FHWA Safety Analyst software tools for identifying an optimal set of safety improvement projects to maximize benefits within a budget constraint.

Dynamic Programming

Dynamic programming methods are used to determine a sequence of interrelated decisions to produce an optimal solution. Dynamic programming problems have a defined beginning and end. While there are multiple intermediate steps between the beginning and end, dynamic programming finds the optimal path between these points. The basic theory of dynamic programming is found by solving a small subset of the original problem, then expanding the subset until the whole problem is considered. The computational complexity of dynamic programming is such that software applications are frequently used for computation. Dynamic programming methods have been used to solve resource allocation problems in Alabama in the past and are currently used for highway safety allocation in Kentucky (Agent, 2003) (Brown, 1990) (AASHTO, Highway Safety Manual, 2010)

4. NONLINEAR SAFETY PROJECT PRIORITIZATION MODEL DEVELOPMENT

The main goal of this study is to formulate a model that will minimize the societal cost associated with vehicular crashes at a defined set of intersections. To do this, the mechanism for crash estimation and reduction at a particular intersection needs to be defined. The methodology established in the HSM completes this task through the application of safety performance functions (SPFs) and crash modification factors (CMFs).

Safety Performance Functions

The HSM uses SPFs as the mechanism for crash estimation. Because crashes are random events, crash frequencies (as shown if figure 3 below) are often an unreliable predictor of the overall safety of a site under consideration.

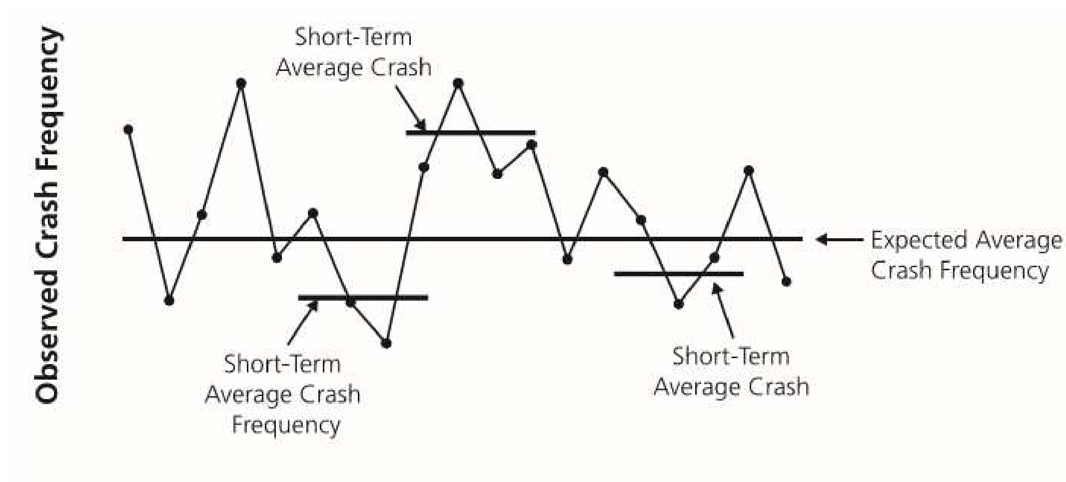

Figure 3. HSM Variation in Short Term Crash Frequency

To account for crash variation, the HSM utilizes SPF's. An SPF is a regression equation that estimates the average crash frequency as a function of segment length and annual average daily traffic (AADT). An example of an SPF for intersections is:

$$N_{spf\ int} = exp[a + b \times \ln(AADT_{maj}) + c \times \ln(AADT_{min})]$$

or

$$N_{spf\ int} = exp[a + d \times \ln(AADT_{total})]$$

Where:

$N_{spf\ int}$ = SPF estimate of intersection-related expected average crash frequency for base conditions;

$AADT_{maj}$ = AADT (vehicles per day) for major-road approaches;

$AADT_{min}$ = AADT (vehicles per day) for minor-road approaches;

$AADT_{total}$ = AADT (vehicles per day) for minor and major-roads combined approaches; and

a, b, c, d = regression coefficients.

SPFs are developed using regression analysis techniques utilizing observed crash data collected at many sites with varying AADTs over many years. To adjust SPFs to account for conditions that are not considered under the base condition, the HSM utilizes Crash Modification Factors (CMFs).

Crash Modification Factors

Crash modification factors are used to quantify the resulting reduction of crashes at a location given the implementation of a particular treatment. The effect of implementing multiple treatments if found by multiplying the predicted crashes at a location by the CMF's as shown in equation (1).

$$N_{Predicted} = N_{SPF} * (CMF_1 * CMF_2 * \ldots * CMF_y) * C \qquad (1)$$

Where:

$N_{Predicted}$ = predicted average crash frequency for a specific year for a given facility

N_{SPF} = predicted average crash frequency determined for base conditions of the SPF (safety performance function) developed for a given facility

CMF = crash modification factors

C = calibration factor to adjust SPF for a given facility to local conditions. This factor will be omitted in the following analysis but in practice can be easily reintroduced.

Crash Weighting Using Empirical Bayes Method

Using observed crash frequency information and crashes predicted from an SPF, expected crashes can be found using the formula

$N_{expected} = w \times N_{predicted} + (1 - w) \times N_{observed}$

Where:

$N_{expected}$ = expected average crashes for the study period

W = weighted adjustment used for SPF prediction

$N_{predicted}$ = predicted average crash frequency predicted using an SPF for the study period under the given conditions.

$N_{observed}$ = observed crash frequency at the site over the study period

The weighted adjustment factor is :

$$w = \frac{1}{1 + k \times \left(\sum_{\substack{\text{all study} \\ \text{years}}} N_{predicted} \right)}$$

Where:

k = overdispersion parameter from the associated SPF

SPFs with a high level of overdispersion are considered less reliable for crash estimation. The weighted adjustment factor places more or less emphasis on the SPFs for situations where overdispersion is low and more emphasis on observed crashes when overdispersion is high.

Societal Cost Estimation

The Federal Highway Administration (FHWA) has quantified the monetary impacts of vehicular crashes for use in analysis. Factors such as property damage, medical care, and lost productivity are taken into consideration when estimating these values. The HSM uses values from the FHWA report Crash Cost Estimates by Maximum Police-Reported Injury Severity within Selected Crash Geometries, detailed in Table 1.

Table 2. Societal Crash Cost Estimates by Crash Severity

Collision Type	Comprehensive Societal Crash Costs	Index Value
Fatal (K)	$4,008,900	0
Disabling Injury (A)	$216,000	1
Evident Injury (B)	$79,000	2
Fatal/Injury (K/A/B)	$158,200	3
Possible Injury (C)	$44,900	4
PDO (O)	$7,400	5

Index values have been created for each collision type for use in the description of the mathematical model. Applying the crash cost information to equation (1), the societal cost of crashes at an intersection is defined as:

$$\text{Societal Cost} = \sum_{k=0}^{4} \left(\prod_{j=1}^{m} CMF_j^k \right) N^k C^k \qquad (2)$$

Where:

N_k= Number of crashes classified with index value k

C_k= Societal cost of crash with index value k

CMF_{jk}= Crash modification factor associated with the implementation of countermeasure j applied to crashes of severity index value k.

For a network of *n* intersections, this results in:

$$\text{Total Societal Cost} = \sum_{i=1}^{n}\sum_{k=0}^{4}\left(\prod_{j=1}^{m} CMF_{j}^{k}\right) N_i^k C^k \qquad (3)$$

This is the objective value to be minimized.

Mathematical Formulation of the Model

To find the optimal set of countermeasures for model implementation, it is necessary to first define a decision variable that will apply the appropriate CMF's.

$$S_{ij} = \begin{cases} 1, & If\ Countermeasure\ j\ is\ selected\ for\ site\ i \\ 0, & Otherwise \end{cases}$$

Incorporating the decision variable into the societal cost expression, the resulting objective function is obtained:

$$\sum_{i=1}^{n}\left[\sum_{k=0}^{4}\left(\prod_{j=1}^{m}\left[CMF_{j}^{k}\ S_{ij}+1-S_{ij}\ \right]\right)\ N_{i}^{k}C^{k}\right] \qquad (4)$$

The manipulation of terms above is necessary to prevent zero values in the product of available CMF's. To prevent countermeasures from being selected at locations where implementation is infeasible or the countermeasure already exists, it is necessary to define:

$$E_{ij} = \begin{cases} 1, & If\ Countermeasure\ j\ exists\ or\ is\ infeasible\ for\ site\ i \\ 0, & Otherwise \end{cases}$$

This provides the means of defining the following constraint:

$$E_{ij} + S_{ij} \leq 1 \qquad (5)$$

The HSM allows a maximum of three countermeasures to be implemented at any location. This creates the following condition:

$$\sum_{j=1}^{m} S_{ij} \leq 3 \quad i = 1, \ldots, n \qquad (6)$$

Finally, the total cost of improvements selected must stay within the prescribed budget.

$$\sum_{i=1}^{n}\sum_{j=1}^{m} S_{ij}P_j \leq B \qquad (7)$$

Where:

B=Allocated budget

P_j= Cost of implementing countermeasure j

Combining (4), (5), (6), and (7), the following model description is obtained:

$$\text{Min} \sum_{i=1}^{n}\left[\sum_{k=0}^{4}\left(\prod_{j=1}^{m}\left[CMF_j^k \; S_{ij} + 1 - S_{ij}\right]\right) \; N_i^k C^k\right]$$

$$\text{s.t. } S_i^j + E_i^j \leq 1 \;\; \forall\, i,j$$

$$\sum_j S_{ij} \leq 3 \;\; i = 1, \ldots, n$$

$$\sum_{i=1}^{n}\sum_{j=1}^{m} S_{ij}P_j \leq B$$

$$S_{ij}, E_{ij} = 0 \text{ or } 1$$

Where:

E_{ij} ={1 if countermeasure j exists or is infeasible for site i, 0 otherwise}

S_{ij} ={1 if countermeasure j is selected for site i, 0 otherwise}

N_{ik} = Number of crashes of severity type k at site i

CMF_{jk} = CMF for severity type k associated with implementation of countermeasure j

C_k =Societal cost associated with a crash of severity k

P_j = Cost of implementation of countermeasure j

B = Construction Budget

n = number of sites under consideration

m = number of available countermeasures

5. CASE STUDY

The intersection data used in this study contain 20 signalized intersections in Reno Nevada. Crash and site inventory data were collected to illustrate a practical application of the model. Observed crashes were used in lieu of expected crashes with understanding that the model can be applied to expected crashes in a straightforward manner.

Figure 4. Site Location Map

Crash Modification Factors

Intersection CMF's and estimated implementation costs are detailed in Table 3. The values chosen are for model illustration on not based on published data.

Table 3. Intersection Treatment Types and Associated Implementation Costs

Countermeasure	PDO Crash CMF	Injury Crash CMF	Fatal Crash CMF	Implementation Cost
Install Left Turn Pocket	0.90	0.90	0.90	$3,000
Install Right Turn Pocket	0.92	0.90	0.90	$3,000
Add Additional Signal Head	0.83	0.83	0.83	$4,000
Install Median	0.73	0.70	0.75	$6,000
Restrict Parking Near Intersection	0.51	0.75	0.75	$15,000

Crash Data

Crash data was collected at the sample set of intersections. Since CMF's were not available for individual injury types, injury crashes were consolidated and represented by a single column. In practice, the most accurate means of processing the data given a single injury crash CMF would be to apply the same CMF to each injury crash type, but maintain the separation of different injury types to apply the appropriate societal costs.

Table 4. Intersection Crash Data

Major Street	Cross Street	PDO Crashes	Injury Crashes	Fatal Crashes
2nd St	Arlington Ave	18	9	1
2nd St	Center St	16	1	0
2nd St	Lake St	7	11	0
2nd St	Sierra St	16	1	0
2nd St	Virginia St	13	6	0
4th St	Arlington Ave	8	17	0
4th St	Center St	9	8	0

Major Street	Cross Street	PDO Crashes	Injury Crashes	Fatal Crashes
4th St	Keystone Ave	23	18	0
4th St	Lake St	6	10	0
4th St	Ralston St	5	9	0
4th St	Sierra St	8	7	0
4th St	Vine St	5	5	0
4th St	Virginia St	8	9	0
4th St	West St	8	3	0
5th St	Center St	6	6	0
5th St	Keystone Ave	14	5	0
5th St	Sierra St	7	6	0
5th St	Virginia St	12	4	0
7th St	Keystone Ave	15	15	0
9th St	Virginia St	24	8	0

Inventory Data

Site visits were performed to investigate the presence/feasibility of locations for implementation of the researched improvement types. Intersection inventory is detailed below, where the value 1 is used where improvement types exist or are infeasible and 0 is used where the improvement does not exist and is feasible for implementation.

Table 5. Site Inventory Data

Major Street	Cross Street	Install Left Turn Pocket	Install Right Turn Pocket	Add Additional Signal Head	Install Median	Restrict Parking Near Intersection
2nd St	Arlington Ave	0	0	1	0	0
2nd St	Center St	0	0	0	1	1
2nd St	Lake St	0	0	1	0	0
2nd St	Sierra St	0	0	1	1	1
2nd St	Virginia St	0	0	0	1	1
4th St	Arlington Ave	0	0	0	0	0
4th St	Center St	1	0	0	0	1
4th St	Keystone Ave	1	1	0	1	1
4th St	Lake St	1	0	1	0	1

Major Street	Cross Street	Install Left Turn Pocket	Install Right Turn Pocket	Add Additional Signal Head	Install Median	Restrict Parking Near Intersection
4th St	Ralston St	1	0	0	0	0
4th St	Sierra St	1	0	1	1	1
4th St	Vine St	0	0	0	0	1
4th St	Virginia St	0	0	0	0	1
4th St	West St	1	0	1	0	0
5th St	Center St	0	0	1	0	1
5th St	Keystone Ave	1	0	0	1	1
5th St	Sierra St	0	0	1	0	1
5th St	Virginia St	0	0	0	0	1
7th St	Keystone Ave	1	1	0	0	1
9th St	Virginia St	0	0	1	0	0

Data Processing

The collected information was applied to the model utilizing a global optimization algorithm provided by Matlab. The algorithm employs a genetic algorithm solver to find the optimization results. The genetic algorithm creates a sample population of solutions for analysis. The algorithm repeatedly modifies the population until it evolves toward an optimal solution (Mathworks, 2017). The model was first tested using 3 allowable countermeasures, then retested using 1 countermeasure. The 3 countermeasure constraint corresponds to the maximum number of allowable countermeasures defined in the HSM, while the 1 countermeasure constraint corresponds to constraints used in common linear formulations such as Safety Analyst (AASHTO, Analytical Tool User's Manual, 2014). The simulation assumed a construction budget of $60,000 and the societal cost values detailed in table 6. The solutions produced by the algorithm are contained in tables 7 and 8. Solutions for both cases were found within 60 generations (as detailed in figures 5 and 6). The script for the code can be found in appendix A.

Table 6. Crash Costs Used In Simulation

Crash Severity	Societal Cost
PDO	$7,000
Injury	$100,000
Fatal	$1,000,000

Table 7. Optimization Results Utilizing 3 Countermeasures

Major Street	Cross Street	Install Left Turn Pocket	Install Right Turn Pocket	Add Additional Signal Head	Install Median	Restrict Parking Near Intersection
2nd St	Arlington Ave	0	0	1	0	0
2nd St	Center St	1	0	0	1	0
2nd St	Lake St	0	0	0	1	0
2nd St	Sierra St	0	0	0	0	0
2nd St	Virginia St	0	0	0	0	0
4th St	Arlington Ave	0	0	0	0	0
4th St	Center St	0	0	1	0	0
4th St	Keystone Ave	0	0	0	0	0
4th St	Lake St	0	0	0	0	0
4th St	Ralston St	0	1	1	1	0
4th St	Sierra St	0	0	0	0	0
4th St	Vine St	0	0	0	0	0
4th St	Virginia St	0	0	0	1	0
4th St	West St	0	0	0	0	0
5th St	Center St	0	1	0	0	0
5th St	Keystone Ave	0	1	0	1	0
5th St	Sierra St	0	0	0	0	0
5th St	Virginia St	0	0	0	0	0
7th St	Keystone Ave	0	0	0	0	0
9th St	Virginia St	0	0	0	1	0

Table 8. Optimization Results Utilizing 1 Countermeasure

Major Street	Cross Street	Install Left Turn Pocket	Install Right Turn Pocket	Add Additional Signal Head	Install Median	Restrict Parking Near Intersection
2nd St	Arlington Ave	0	0	1	0	0
2nd St	Center St	0	0	0	1	0
2nd St	Lake St	0	0	0	0	0
2nd St	Sierra St	0	0	0	0	0
2nd St	Virginia St	0	1	0	0	0
4th St	Arlington Ave	0	0	0	0	0
4th St	Center St	0	0	1	0	0
4th St	Keystone Ave	0	0	0	0	0
4th St	Lake St	0	0	0	1	0
4th St	Ralston St	0	0	1	0	0
4th St	Sierra St	0	0	0	0	0
4th St	Vine St	0	1	0	0	0
4th St	Virginia St	0	1	0	0	0
4th St	West St	0	0	0	1	0
5th St	Center St	0	0	0	1	0
5th St	Keystone Ave	1	0	0	0	0
5th St	Sierra St	0	0	0	0	0
5th St	Virginia St	0	1	0	0	0
7th St	Keystone Ave	0	1	0	0	0
9th St	Virginia St	0	0	0	1	0

4. DISCUSSION OF RESULTS

Implementation of multiple countermeasures produces significantly better results. With the ability to implement 3 countermeasures, the total societal benefit for the simulation was $3,481,100. When limited to a single countermeasure, the resulting societal benefit was $2,950,050. While the single countermeasures converged slightly quicker, the processing time was less than one minute for both models.

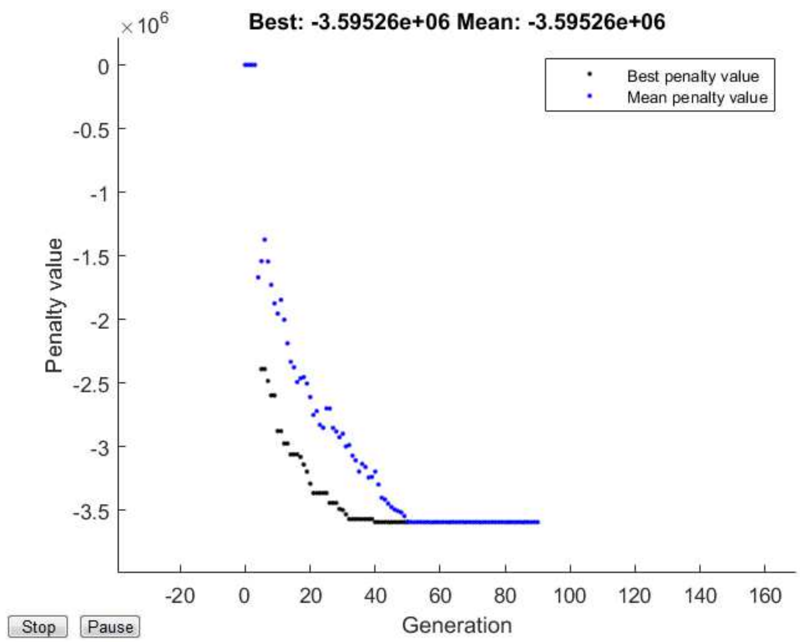

Figure 5. Survival Function Graph for 3 Countermeasures

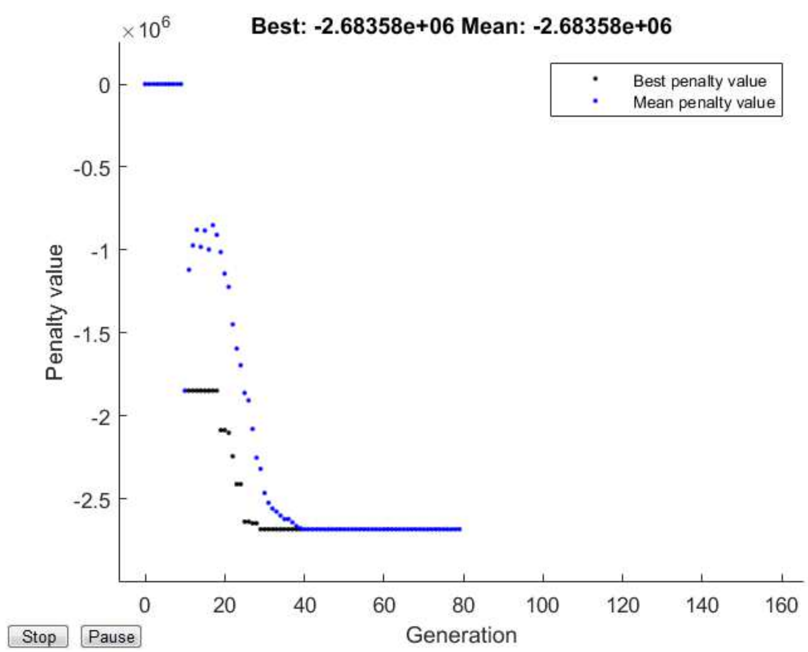

Figure 6. Survival Function Graph for 1 Countermeasure

5. CONCLUSION

Though more computationally complex, the nonlinear model offers significant advantages over linear models. The method presented in this paper neglects certain factors, such as the service life of the available improvements. A modified analysis containing these impacts can be performed in a straightforward manner. Also, the assumption used throughout the development of the model is that CMF's can be combined by multiplication. As our understanding of the interaction between various countermeasures evolves, this may prove to not be the case. As long as the effects of implementing multiple countermeasures involve products of individual reductions, nonlinear programming will still provide the most suitable treatment for this type of problem.

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APPENDIX A – MATLAB CODE

OBJECTIVE FUNCTION DEFINITION

```
function objectiveFunction = objectiveFunctionCountermeasure( x )
load('matlab.mat')
x_matrix=reshape(x,[20,5]);
solutionSize=size(x_matrix);

if solutionSize(1)==20 && solutionSize(2)==5
%     sumRow=sum(x,2)
%     for i=1:20
%         if sumRow(i,1)>3
%             objectiveFunction=0;
%             return;
%         end
%     end
    totalBenenifit=0;
    for i=1:20
        totalCMFPDO=1;
        totalCMFInjury=1;
        totalCMFFatal=1;
        for j=1:5
            Ele=x_matrix(i,j);
            if Ele==1
                totalCMFPDO=totalCMFPDO*CMF(1,j);
                totalCMFInjury=totalCMFInjury*CMF(2,j);
                totalCMFFatal=totalCMFFatal*CMF(3,j);
            end
        end
        reducedPDO=SPF(i,1)*(1-totalCMFPDO);
        reducedInjury=SPF(i,2)*(1-totalCMFInjury);
        reducedFatal=SPF(i,3)*(1-totalCMFFatal);

benefit=reducedPDO*SocietalCost(1,1)+reducedInjury*SocietalCost(2,1)+reducedFatal*Societal
Cost(3,1)
        totalBenenifit=totalBenenifit+benefit;
    end
    objectiveFunction=-totalBenenifit;
else
    objectiveFunction=0;
end
%UNTITLED2 Summary of this function goes here
%   Detailed explanation goes here
objectiveFunction

end
```

CONSTRAINT DEFINITIONS

```
function [c,ceq]= simple_constrain( x )
%UNTITLED4 Summary of this function goes here
%   Detailed explanation goes here
load('matlab.mat')
x_matrix=reshape(x,[20,5]);
ceq=[];
solutionSize=size(x_matrix);
% if solutionSize(1)==20 && solutionSize(2)==5
sumRow=sum(x_matrix,2)
% constrain=max(sumRow)-3;
constrain=max(sumRow)-1;
%     for i=1:20
%         if sumRow(i,1)>3
%             constrain=sumRow(i,1)-3;
%             break;
%         end
%     end

totalCost=0;
for i=1:20
    for j=1:5
        Ele=x_matrix(i,j);
        if Ele==1
            totalCost=totalCost+countermeasureCost(1,j);
        end
    end
end
costDifference=totalCost-budget
c=[constrain costDifference]
% end
%  c=[-1;-1]
end
```

SAMPLE DATA

```
function [c,ceq]= simple_constrain( x )
%UNTITLED4 Summary of this function goes here
%   Detailed explanation goes here
load('matlab.mat')
x_matrix=reshape(x,[20,5]);
```

```
ceq=[];
solutionSize=size(x_matrix);
% if solutionSize(1)==20 && solutionSize(2)==5
sumRow=sum(x_matrix,2)
% constrain=max(sumRow)-3;
constrain=max(sumRow)-1;
%     for i=1:20
%         if sumRow(i,1)>3
%             constrain=sumRow(i,1)-3;
%             break;
%         end
%     end

totalCost=0;
for i=1:20
    for j=1:5
        Ele=x_matrix(i,j);
        if Ele==1
            totalCost=totalCost+countermeasureCost(1,j);
        end
    end
end
costDifference=totalCost-budget
c=[constrain costDifference]
% end
%  c=[-1;-1]
end
```

OPTIMIZATION CODE

```
function optimizedCount = crashCounterOptimization( )
%UNTITLED3 Summary of this function goes here
%   Detailed explanation goes here
load('matlab.mat')
% problem = createOptimProblem('fmincon','x0',NewCount,'objective',@objectiveFunctionCountermeasure,'lb',LB,'ub',UB);
% fmincon(problem);
nvars=100;
options = gaoptimset('TolCon',1e-6,'TolFun',1e-12);
% options=gaoptimset('PopulationSize',200);
% options = optimoptions(options,'PlotFcn',{@gaplotbestf,@gaplotmaxconstr},'Display','iter');
[x,fval] = ga(@objectiveFunctionCountermeasure,nvars,[],[],[],[],LB,UB,@simple_constrain,1:100
```

```
)
x_matrix=reshape(x,[20,5])
fval
%                                   x                                   =
fmincon(@objectiveFunctionCountermeasure,NewCount,[],[],[],[],LB,UB,@simple_cons
train)
end
```