

# Learning And Teaching Fractions At The Elementary Level

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**Abstract:** The concept of fractions is often thought of as one of the most difficult mathematical topics taught in elementary school, but why do students find fractions so confusing? It may be because fractions can represent so many different things – a part of a whole, a division, or a point on a number line. Or because students' knowledge of whole numbers makes it difficult to understand why one half is larger than one-third. It also maybe because their teachers do not have a deep enough understanding of the subject. It may be a combination of these things. Whatever the reasons, the result of all of this confusion is often that fractions are taught in a less than meaningful way. Students learn vocabulary and quick tricks rather than what a fraction means. Teachers themselves do not have a complete understanding -- focus on one fraction model and use only one type of manipulative, if they use manipulatives at all, to instruct their students. These students may be able to successfully answer standardized test questions but do not understand fractions. Because they are an important concept in both school and everyday life, students need to develop a deeper understanding of fractions in an environment that supports investigation and inquiry.

**Keywords:** Fractions, Elementary level fractions, Learning fractions, Teaching fractions, Fraction model

## Review of the Literature

Researchers have studied many aspects of mathematics education. This literature review explores how elementary students engage in learning about fraction concepts, elementary teachers' understanding of fraction concepts, and how the quality of questioning affects student understanding. The teaching and learning of mathematics, like any other subject, requires that both the teacher and learner communicate effectively. In Halliday's (1975) view, learning a language involves 'learning how to mean'. Thus, the language of mathematics involves learning how to make and share mathematical meanings using language appropriate to the context, which is more than recognizing and responding to words in isolation. Starting in first grade, teachers introduce the beginning concepts of fractions by showing picture representations for wholes, halves, and fourths. This transfers to labeling fraction models. By fifth grade, students are expected to multiply and divide fractions. They must be able to comprehend what fractions mean before they can manipulate fractions through various operations and within word problems. Taking the vocabulary back to a strategy that emphasizes oral language development can help improve students' comprehension of the meaning of fractions. This, in turn, demands the use of appropriate language (words and symbols) whose level of difficulty is at par with the cognitive abilities of the learners concerned. Communicating mathematical ideas so that the message is adequately understood is difficult enough when the teacher and learner have a common first language, but the problem is acute when their preferred languages differ. Several studies have indicated that a student's command of English plays a role in his/her performance in mathematics. Souviney (1983) evaluated students in grades 2, 4, and 6 with various languages and mathematics instruments on eight measures of cognitive development. His results showed that English reading and Piagetian measures of conservation were highly correlated with mathematical achievements. The primary function of language, in mathematics instruction, is to enable both the teacher and the learner to communicate mathematical knowledge with precision. To realize the

objectives of mathematics instruction, teachers and textbook authors need to use a language whose structure, meaning, technical vocabulary, and symbolism can be understood by learners of a particular class level. The communication of meaning frequently involves interpretation on the part of the receiver and this should warn us that messages could be given incorrect interpretations. Donaldson (1978) suggested that:

When a child interprets what we say to him, his interpretations are influenced by at least 3 things- his knowledge of the language, his assessment of what we intend (as indicated by our non-linguistic behavior), and how he would represent the physical situation to himself. Some of the words and symbols used to communicate mathematical ideas can sometimes be misinterpreted by learners in their attempt to imitate their teachers. Pimm (1987 as cited in Muhandiki, 1992) reported that apart from determining the patterns of communication in the classroom, the teacher also serves as a role model for a 'native speaker' of mathematics. Hence the learners' search for the meaning of whatever they hear can, sometimes, lead to wrong conclusions. An instance of the learners' tendency to change (though not deliberately) the meaning of mathematical words into what they think the teacher intended to say is reported in Orton (1987) as follows:

A kindergarten teacher drew a triangle, a square, and a rectangle on the blackboard and explained each to her pupils. One little girl went home, drew the symbols, and told her parents: 'this is a triangle, this is a square and this is crashed angle' This observation shows that the little girl's interpretation of 'rectangle' as 'crashed angle' exemplifies a situation whereby the child has a correct symbolic representation of a concept whose technical term she cannot produce due to linguistic problems. It is, however, important for teachers to realize that the process of learning definitions of mathematical terms can be complicated by the abstract nature and the consequent difficulty of the words used to refer to them. Since students can find it difficult to comprehend the meaning of some terms even after they have

been defined, the teacher ought to discuss various meanings and interpretations of such words and phrases so that each becomes aware of what the other means and understands by particular linguistic forms. Further, Dickson, Brown, and Gibson, (1984), have asserted that many specialized terms have an essential and rightful place in mathematics, and it is necessary to incorporate them into the learning and teaching of the subject. From the foregoing, it can be seen that language is critical to many of the processes of learning and instruction, and it confers many benefits in terms of enabling us to articulate, objectify and discuss the problems which the field of mathematics presents. Yet language brings its own rules and demands, which are not always in perfect correspondence with the rules and demands of mathematics; it presents ambiguities and inconsistencies which can mislead and confuse.

### **Pre-service Teachers' Understanding of Fraction Concepts**

Because pre-service teachers will be instructing the students at the STEM Fraction Learning Station, their attitudes and knowledge about both teaching and learning mathematics, in general, and about fractions is an important aspect of this research. "Prospective teachers themselves are successful graduates of schools as they are now, with mathematics classrooms that tend to focus on the learning and application of routine and procedural skills" (Nicol, 1999, p.45). As a result, studies have shown that pre-service, elementary teachers have limited knowledge of fraction concepts (Newton, 2008). To teach fractions, a notoriously difficult subject, to their students, teachers need to have more than a surface understanding. Students learn more when their teachers know more. Over a semester, Newton (2008) studied undergraduate, pre-service teachers' understanding of addition, subtraction, multiplication, and division of fractions and their ability to solve fraction problems efficiently and flexibly. The 88 undergraduate students were enrolled in a mathematics methods course designed to promote mathematics understanding, and they were given a pre-and post-test to measure their understanding. At the beginning of the semester, students "demonstrated limited and fragmented knowledge of fractions" (p.1104). Their work revealed their misconceptions about fraction concepts and showed that they often inappropriately applied memorized fraction rules and procedures to solve problems. They also demonstrated little flexibility in their ability to solve problems. At the end of the university course, though, which "explicitly linked fraction concepts and procedures," pre-service teachers performed better and demonstrated a deeper understanding of fractions (p.1104). Often, pre-service teachers are taught to teach mathematics in a way that they have never actually experienced; this is a "tremendous challenge for teacher education" (Nicol, 1999, p.46). Based on Newton's research, pre-service teachers would benefit greatly from experiencing the type of teaching that they did not experience as children.

### **Learning Fractions in Elementary School**

If pre-service teachers – adults – struggle with fraction concepts, it is not surprising that fractions are a difficult topic for many children (Aksu 1997, Cramer, Post and Delmas 2002, Hasemann 1981). "Only 42% of fourth graders in the NAEP sample could choose a picture that represented a fraction equivalent to a given fraction, and only 18% could

shade a rectangular region to represent a given fraction" (Cramer, Post, and Delmas, 2002, p. 112). Often, children learn fraction rules without any conceptual understanding of fractions (Aksu 1997, Brown and Burton 1978, Hasemann 1981). Peck and Jencks (1981) interviewed hundreds of students to learn about their understandings of fractions and then chose 20 sixth-grade students at random; they found that "about 55% of the students interviewed were unable to demonstrate that they possessed a meaningful concept of a fraction" (p.347). Students appeared to be using rules to go "through the motions" in their attempts to work with fractions (p.348). Hasemann (1981) found that when students did have a rule available to them, they usually "mechanically" used it "sometimes with success but sometimes with nonsensical results" (p.81). Aksu (1997) stated that a common mistake in the teaching of fractions is expecting students to compute fractions before they understand the meaning of fractions. Cramer, Post, and Delmas (2002) compared the effects of a commercial fraction curriculum - which focused on rules, computation, and procedures - and the Rational Number Project (RNP) curriculum - which emphasized understanding through the use of various concrete models. The fourth- and fifth graders that were taught using the RNP curriculum had higher average scores on the post-test, and in post-learning interviews, they were able to approach fraction problems conceptually by "building on their mental images" (p.138). Hallett, Nunes, and Bryant (2010) researched how children make use of their conceptual and procedural knowledge and found that children combine conceptual and procedural knowledge about fractions in unique ways and with varying success. "Some children rely more on concepts, some rely more on procedures, and some rely on both, there are two types of children who struggle with fractions: one group that has problems with conceptual knowledge and one group that has problems with procedural knowledge" (p. 404). The authors conclude that the most successful children can combine their knowledge of procedure and concept, but problem-solving approaches that relied more heavily on conceptual knowledge were usually more successful than those that relied too heavily on procedural knowledge. While both conceptual and procedural knowledge is important, the literature shows that conceptual knowledge should be the basis of fraction learning with procedures introduced later. "The Principles and Standards for School Mathematics support the notion that Grades 3-5 are the critical years for developing a solid conceptual framework" (Cramer, Post, and Delmas, 2002, p. 139).

### **Mathematical Manipulatives**

How can teachers create a learning environment in which students can develop an appropriate conceptual base for learning fractions? Manipulatives - physical objects used to support mathematical learning - are one way for students to develop a conceptual understanding of fractions, and they are becoming increasingly popular in the elementary classroom. Manipulatives are "objects designed to represent explicitly and concretely mathematical ideas that are abstract. They have both visual and tactile appeal and can be manipulated by learners through hands-on experiences" (Moyer, 2001, p.176). The popularity of the use of manipulative materials stems, in part, from the influential ideas of cognitive developmentalists like Jean Piaget (1971) who proposed that children construct knowledge through action and believed

that young students needed experience with concrete objects as they were not yet mature enough to work abstractly. The National Council of Teachers of Mathematics (1989) stated that children come to mathematical understandings through “classroom experiences in which [they] first manipulate physical objects” and eventually progress to working with symbols that are “meaningfully linked to concrete materials.” Various studies have shown that the use of manipulatives improves children’s learning of mathematics by helping them develop an understanding (Uttal, Scudder and DeLoache 1997, Sowell 1989, Manches, O’Malley and Benford 2010, Raphael and Wahlstrom 1989, Mistretta and Porzio, 2000). “Both teachers and researchers have suggested that concrete objects allow children to establish connections between their everyday experiences and their nascent knowledge of mathematical concepts and symbols ... [they] provide a way around the opaqueness of written mathematical symbols (Uttal, Scudder, and DeLoache, 1997, p.38). Marshall and Paul (2008) found that 95% of teachers surveyed believe that “mathematical manipulatives enhance children’s learning” (p.344). Hasemann (1981) believes that children “can only develop a relational understanding [of fractions] by using concrete materials and diagrams before forming mathematical concepts” (p.83). Despite evidence of the positive effects of manipulatives, their use varies across grade levels. Mathematical manipulatives are more prevalent in the primary grades than they are in the intermediate grades; manipulative use decreases as grade level increases (Uribe-Florez and Wilkins 2010, Marshall, and Paul 2008, Malzahn 2002). Using data from national survey results, Malzahn (2002) found that 57% of K-2 classes used concrete materials in math lessons while only 15% of 3-5 classes did so. Similarly, Uribe-Florez and Wilkins (2010) found that manipulatives were used most often in kindergarten and least often in grades 3-5. “Reasons for this reduction in manipulative use may relate to increased use of textbooks, a view that using manipulatives is ‘babyish,’ or a lack of awareness of how manipulatives may be used to develop mathematical concepts with older children” (Marshall and Paul, 2008, p. 345). Moyer (2001) found that many teachers used manipulatives only as a reward or a fun activity when there is extra time; they did not regard manipulative use as real math. Not surprisingly, textbook/worksheet and manipulative use follow opposite trends; unlike manipulatives, as grade level increases textbook and worksheet work become more frequent (Malzahn 2002). Though manipulative use has positive effects on learning, concrete objects alone are not the answer to the problem of conceptual understanding of mathematics, especially regarding such a complicated topic as fractions. “Although kinesthetic experience can enhance perception and thinking, understanding does not travel through the fingertips and up the arm” (Ball, 1992, p.47). Children do not learn fractions simply by picking up a manipulative. Manipulatives are meant to be used as a tool; the “concreteness of the objects does not, in itself, hold the key to unlocking the mysteries of mathematics” (Uttal, Scudder, and DeLoache, 1997, p.50). Teachers must teach and students must understand that the manipulatives they are working with are a representation of an abstract mathematical concept. The use of math manipulatives in teaching mathematics has a long tradition and solid research history. Manipulatives not only allow students to construct their cognitive models for abstract mathematical ideas and processes, but they also provide a

common language with which to communicate these models to the teacher and other students. In addition to the ability of manipulatives to aid directly in the cognitive process, manipulatives have the additional advantage of engaging students and increasing both interest in and enjoyment of mathematics by building up a strong base of the mathematical language. For example, Base Ten Blocks are constructed in powers of ten, representing ones, tens, hundreds, and thousands. The materials include 1-centimeter unit cubes to represent ones, 10-centimeter rods to represent tens, and 10-centimeter square blocks to represent hundreds. They can be used to teach fractions as well. Though adults may be able to see the connection between the objects and the mathematics, Uttal, Scudder, and DeLoache (1997) suggest that “the relation between manipulatives and their intended referents may not be transparent to children” (p.44). Therefore, teachers must take the time to emphasize these relationships so they can foster an environment in which students’ mathematical development progresses from the concrete to the abstract. When students do not see the relationship between the manipulatives and the symbols, they may “use manipulatives in the same unthinking ways that they use algorithms” (Ambrose, 2002, p.20).

### Learning in Small Groups

Students who participate in stations at the STEM Studio do so in small, cooperative groups. The literature has shown that small group situations affect learners in a variety of ways. Researchers have found much evidence to suggest that cooperative learning has a positive impact on children (Blumenfeld et al. 1996, Lampe, Rooze and Tallent-Runnels 1996, Vaughan 2002, Gillies 2004, Peterson, and Miller 2004). “Peer learning has been suggested by many as an educational innovation that can transform students’ learning experiences improve attitudes toward school, foster achievement, develop thinking skills, and promote interpersonal and intergroup relations” (Blumenfeld et al. 1996). Peterson and Miller (2004) studied college students and found that students engaged in small group learning were more motivated and more likely to be on task than students engaged in large-group instruction. Lampe, Rooze, and Tallent-Runnels (1996) found that cooperative learning increased the achievement of Hispanic 4th-grade students in the social studies curriculum. Similarly, Vaughan (2002) found that cooperative learning increased achievement and attitude about mathematics for students of color. Because of the demographics of the students that visit the STEM Studio, these two studies have particular relevance for the current research. In the same way, manipulative materials rely on other factors to be successful teaching tools, simply arranging students in small groups and sending them off to work is not an appropriate use of cooperative learning. Teachers must create the conditions for successful small-group work; in fact, they “play a vital role in setting up the conditions for collaborative learning” (Mueller and Fleming, 2001, p.265). Listening, asking questions that scaffold thinking, and providing support are essential aspects of the teacher’s role in small-group work (Muller and Fleming, 2001). In the STEM Studio, pre-service teachers involved with this station will be fulfilling all of these roles. Teachers must group students in ways that accommodate learning. Gillies (2004) found that structured groups were more successful than unstructured groups. Though students are not intentionally grouped in the STEM Studio, I believe that the



constant presence of two pre-service teachers will negate any effects of an unstructured group.

### Teacher Questioning

Studies have shown that classroom teachers ask approximately 300-400 questions per day (Levin and Long 1981). Questioning is an essential aspect of teaching and learning, especially in the case of mathematics and the difficult subject of fractions. "Above all," says Donald Fairbairn (1987), "a good teacher knows how to ask the right questions at the right time" (p.19). The right questions can have a tremendous positive impact on student learning (Vogler 2005, Napell 1978, Fairbairn 1987). Good questions promote thought and encourage expression; they allow students to become active participants in their learning. Good questions allow students to "think about the content being studied, connect it to prior knowledge, consider its meanings and implications, and explore its applications" (Vogler, 2005, p. 98). Unfortunately, however, many teachers struggle to ask effective questions (Vogler 2005, Napell 1978, Fairbairn 1987, Nicol 1999). Kenneth Vogler believes that to ask effective questions, teachers must have an understanding of question taxonomies. Bloom's Taxonomy is the most popular and widely known of the questioning taxonomies. Benjamin Bloom identified six levels of questions: knowledge, comprehension, application, analysis, synthesis, and evaluation. Questions that "have only one 'correct' answer and require only minimal mental activity" are at the knowledge end of Bloom's while "more complex questions requiring greater mental activity" are at the evaluation end. (Vogler, 2005, p.98). Knowledge questions require students to simply recall information they have memorized. Comprehension questions require students to demonstrate understanding of knowledge by comparing and contrasting or finding the main idea, among other things. Application questions require students to use their knowledge to solve a problem. Analysis questions involve the investigation of a concept, and synthesis questions require students to use their knowledge to create something new. Questions that address evaluation, the highest of Bloom's levels, require that students use their knowledge to make value judgments. Sandra Napell (1978) claims, "instructors [often] limit their classroom and examination questions to the lowest intellectual levels by demanding only that the students recall the appropriate information" (p.193). If teachers do not have a significant understanding of the diverse levels of questions, it is possible, says Vogler, that they may disservice their students by continually asking only low-level questions and ignoring higher-level tasks completely. No matter what the level, the quality of a question will determine how students learn and think about a topic. High-quality questions at every Bloom's level are important. Sondra Napell (1978) identified five types of questions that should be avoided because they "confuse thinking and suppress responses" (p.188). They are dead-end or yes-no questions, chameleon or run-on questions, the question with a programmed answer, the put-down question, and the fuzzy question. Yes-no questions are often asked in an attempt to assess understanding, but what does a yes or a no-tell a teacher about what her students understand? Yes-no questions should be reworded to allow students to explain their ideas and thinking, these explanations allow teachers to gain a far deeper insight into their students' understanding. Chameleon or run-on questions are "a series of questions asked virtually in one

breath" (Napell, 1978, p.190). Not only do run-on questions deny students "time to reflect and formulate answers," often, but each question of the series also is a little bit different from the last, and students become increasingly unsure of what is being asked and how they should answer (Napell, 1978, p.190). As a result, they are dissuaded from volunteering any answer. Questions with a programmed answer are questions that do not allow for student thought or expression; instead, they lead students to a desired, correct response. Put-down questions do little to create a positive classroom community because they discourage students from asking their questions or volunteering their thoughts. Lastly, fuzzy questions are far too vague for students to formulate high-level responses. Donald Fairbairn (1987) identified the seven deadly sins of questioning; these are seven types of questions that he believes should be eliminated from the classroom. They are yes-no questions, overlaid or multiple questions, ambiguous questions, leading questions, chorus response questions, whiplash questions, and teacher-centered questions. The first four question categories – yes-no, multiple, ambiguous, and leading – are similar to Napell's yes-no, run-on, fuzzy, and programmed answer questions. Chorus response questions are questions that the entire class answers together. These questions are harmless, explains Fairbairn, but should not be asked because they serve no purpose, as they do nothing to increase student understanding. Whiplash questions are a combination of a statement and a question. For example, "The red trapezoid is what?" Statements that are unexpectedly turned into questions are confusing for students. Teacher-centered questions – If I have one trapezoid, what fraction do I have? For example, make it appear to the student that they are responding "to help the teacher rather than to clarify their knowledge" (Fairbairn, 1987, p.20). There are certainly many opportunities for teachers to ask bad questions. In addition to discussing what makes a bad question, Donald Fairbairn (1987) discusses the qualities of good questioning. Good questions are planned, consider the instructional level of the students, and involve all students. They should be direct and clear, logically sequenced, and require deep thinking. Additionally, teachers should increase wait time – the time between asking a question and getting a response. Studies have shown that increased wait time is one of the most effective ways of involving a larger number of students in a lesson and eliciting well-thought-out responses (Fairbairn 1987). Fairbairn (1987) recommends that teachers "not talk so much" and instead, let students do most of the talking (p.22). There are many different perspectives on the most effective way to teach mathematics. Based on the less-than-stellar performance of students in the area of fractions, it is clear that students need far more opportunities to form conceptual understandings of the topic.

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